Spin-dependent transmission through a chain of rings: influence of a periodically modulated spin-orbit interaction strength or ring radius

B. Molnár, P. Vasilopoulos,* and F. M. Peeters

Departement Natuurkunde, Universiteit Antwerpen (Campus Drie Eiken), B-2610 Antwerpen, Belgium

We study ballistic electron transport through a finite chain of quantum circular rings in the presence of spin-orbit interaction of strength α. For a single ring the transmission and reflection coefficients are obtained analytically and from them the conductance for a chain of rings as a function of α and of the wave vector k of the incident electron. We show that due to destructive spin interferences the chain can be totaly opaque for certain ranges of k the width of which depends on the value of α. A periodic modulation of the strength α or of the ring radius widens up the gaps considerably and produces a nearly binary conductance output.

In recent years the study of spintronics devices, which utilize the spin rather than the charge of an electron, has been intensified mainly because they are expected to operate at much higher speeds than the conventional ones and have potential applications in quantum computing. One such device is a single ring in the presence of the Rashba coupling[1] or spin-orbit interaction (SOI) which results from asymmetric confinement in semiconductor nanostructures. It is important in materials with a small band gap such as InGaAs. An important feature of electron transport through a ring is that, even in the absence of an external magnetic field, the difference in the Aharonov-Casher phase of carriers travelling clockwise and counterclockwise produces spin-sensitive interference effects.[2] Here we build on this fact by studying electron transport through a chain of identical rings in the presence of SOI of strength α. In addition, we study the influence of periodic modulations of α or of the ring radius. The main motivation behind this study is to produce a controllable, binary output, with wide gaps, upon periodically varying α and/or ring radius. Such binary output is pertinent to the development of the spin transistor.[?] Here are the context and the results.

Single-ring transport. In the presence of SOI the Hamiltonian operator for a one-dimensional ring reads[4]

$$H = \hbar\Omega\big[-i\frac{\partial}{\partial\varphi} + \frac{\omega_{so}}{2\Omega}(\cos\varphi\sigma_x + \sin\varphi\sigma_y)\big]^2 \qquad (1)$$

where σ_x, σ_y, and σ_z are the Pauli matrices. The parameter Ω denotes $\hbar/2m^*a^2$ and $\omega_{so} = \alpha/\hbar a$ is the frequency associated with the SOI. The parameter α represents the average electric field along the z direction. For an InGaAs-based system α can be controlled by a gate voltage with values in the range $(0.5-2.0)\times 10^{-11}$eVm.[5]

Eigenvalues, eigenfunctions. The energy spectrum $E_n^{(\mu)}$ and unnormalized eigenstates $\Psi_n^{(\mu)}$ pertaining to Eq. (1) are labeled by the index $\mu = 1,2$ and given by[6]

$$E_n^{(\mu)} = \hbar\Omega(n - \Phi_{AC}^{(\mu)}/2\pi)^2,\ \Psi_n^{(\mu)}(\varphi) = e^{in\varphi}\chi^{(\mu)}(\varphi); \qquad (2)$$

the orthogonal spinors $\chi^{(\mu)}(\varphi)$ can be expressed in terms of the eigenvectors $[1,0]^T$, $[0,1]^T$ of the matrix σ_z as

$$\chi^{(1)} = [\cos\frac{\theta}{2}, e^{i\varphi}\sin\frac{\theta}{2}]^T, \chi^{(2)} = [\sin\frac{\theta}{2}, -e^{i\varphi}\cos\frac{\theta}{2}]^T \qquad (3)$$

with $\chi^{(\mu)} \equiv \chi^{(\mu)}(\varphi)$, T denoting the transpose of the row vectors, and $\theta = 2\arctan[\Omega/\omega_{so} - (\Omega^2/\omega_{so}^2+1)^{1/2}]$. The spin-dependent term $\Phi_{AC}^{(\mu)}$ is the Aharonov-Casher phase $\Phi_{AC}^{(\mu)} = -\pi[1+(-1)^\mu(\omega_{so}^2+\Omega^2)^{1/2}/\Omega]$.

The ring connected to two leads is shown in the inset to Fig. 1 with the local coordinate systems attached to the different regions. The appropriate boundary conditions to apply at the intersections are a spin-dependent version of Griffith's boundary conditions.[6] Specifically, at each junction the wave function must be continuous and the spin probability current density must be conserved.

In the present problem the total wave function in the lead can be expanded in terms of spinors $\chi^{(\mu)}$. We have

$$\Psi_I(x) = \sum_{\mu=1,2}[e^{ikx}f^{(\mu)} + e^{-ikx}r^{(\mu)}]\chi^{(\mu)}(\pi), \qquad (4)$$

$$\Psi_{II}(x') = \sum_{\mu=1,2}(-1)^{\mu+1}[e^{ikx'}t^{(\mu)} + e^{-ikx'}g^{(\mu)}]\chi^{(\mu)}(0) \qquad (5)$$

in region I and II, respectively, and k denotes the incident wave vector. The coefficients $f^{(\mu)}$ ($g^{(\mu)}$) are the amplitudes of the spin state $\mu = 1,2$ for electrons incident from the left (right) lead and $r^{(\mu)}$ ($t^{(\mu)}$) those of the spin state which are reflected to the left (exiting to the right) of the ring. One can show that the spinor $\chi^{(\mu)}(0) = (-1)^{\mu+1}U\chi^{(\mu)}(\pi)$, with the unitary operator U having the form $U = \begin{bmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{bmatrix}$. A similar expansion can be made for the wave functions in the upper and lower arms of the ring. The result is

$$\Psi_{up}(\varphi) = \sum_{\mu,j=1}^{2} a_j^{(\mu)}e^{in_j^\mu\varphi}\chi^{(\mu)}(\varphi), \qquad (6)$$

$$\Psi_{low}(\varphi') = \sum_{\mu,j=1}^{2} b_j^{(\mu)}e^{-in_j^\mu\varphi'}\chi^{(\mu)}(-\varphi'), \qquad (7)$$

with $n_j^\mu = (-1)^j ka + \Phi_{AC}^{(\mu)}/2\pi$ the solutions of the equation $k^2a^2 = E_{n^\mu}^\mu/\hbar\Omega$ that ensure energy conservation.

Reflection and transmission coefficients. Applying the boundary conditions one can verify that the amplitudes $g^{(\mu)}$ and $t^{(\mu)}$ are connected to $r^{(\mu)}$ and $f^{(\mu)}$ by a transfer

matrix L independent of the spin index μ as

$$L[r^{(\mu)}, f^{(\mu)}]^T = [g^{(\mu)}, t^{(\mu)}]^T. \quad (8)$$

The matrix L above can be written in an analytic form

$$L = (1/T)\begin{bmatrix} 1 & -R \\ R & (T)^2 - (R)^2 \end{bmatrix}, \quad (9)$$

where T and R are the functions of ka and $\Delta_{AC} = (\Phi^{(1)}_{AC} - \Phi^{(2)}_{AC})/2$ (hence of SOI strength α)

$$T = \frac{i\sin(\Delta_{AC}/2)\sin(ka\pi)}{\sin^2(\Delta_{AC}/2) - [\cos(ka\pi) - i\sin(ka\pi)/2]^2}, \quad (10)$$

$$R = \frac{[1 + 3\cos(2ka\pi) + 4\cos\Delta_{AC}]/8}{\sin^2(\Delta_{AC}/2) - [\cos(ka\pi) - i\sin(ka\pi)/2]^2}. \quad (11)$$

Let us assume that an electron enters the ring from the left with an arbitrary spin orientation ($f^{(\mu)}$ are arbitrary complex numbers) but $g^{(1)} = g^{(2)} = 0$, i. e., that there is no incident electron current from the right. Then the electron is reflected without changing its original spin-orientation with the probability amplitude $R = -L_{12}/L_{11}$; on the other hand, it is transmitted with the transmission coefficent $T = -L_{12}L_{21}/L_{11} + L_{22}$ but its spin is unitarily rotated by U. One can show that the standard relation $|T|^2 + |R|^2 = 1$ is held.

Multi-ring conductance. If we have N rings, the single-ring result can be easily generalized for a chain if the rings only touch each other, cf. inset of Fig. 3(d1). First, one has to calculate the joint transfer matrix $\tilde{L}$

$$\begin{bmatrix} g^{(\mu)}_N \\ t^{(\mu)}_N \end{bmatrix} = \tilde{L}\begin{bmatrix} r^{(\mu)}_1 \\ f^{(\mu)}_1 \end{bmatrix} = L_N ... L_1 \begin{bmatrix} r^{(\mu)}_1 \\ f^{(\mu)}_1 \end{bmatrix}, \quad (12)$$

then apply the boundary condition $g^{\mu}_N = 0$, that is, after the last ring there is only outgoing wave function. Then the reflection ($\widetilde{R}$) and transmission ($\widetilde{T}$) coefficients are written in terms of $\tilde{L}$ and the conductance G reads

$$G = 2(e^2/h)\left|\widetilde{T}\right|^2 = 2(e^2/h)\left|\tilde{L}_{12}\tilde{L}_{21}/\tilde{L}_{11} - \tilde{L}_{22}\right|^2. \quad (13)$$

We note that the spin of the electrons exiting the chain is rotated by $\tilde{U} = U(N)..U(1)$ in respect to that of the incident electrons.

Numerical results. In Fig. 1 the conductance $G(ka)$, through a chain of $N = 101$ rings, is shown as a function of the incident wave vector ka for various values of α. Because $G(ka)$ is an even and periodic function of ka with period 1 we show it only within one period of ka, for $5 \le ka \le 6$. In the absence of SOI the conductance G oscillates with high values but it never drops to zero. In other words, a chain without SOI is *never* totally reflexive. But for finite non zero values of α it takes a "square-wave" form and has *zero* value in a *finite* range of k, the width of which strongly depends on α. Outside these ranges G always oscillates with high values.

The occurence of the gaps in $G(ka)$ is attributed to the Aharonov-Casher effect due to the spin precession induced by the SOI and the destructive interference between the electron spins travelling in the clockwise and counter-clockwise directions. In the most extreme cases, if the SOI strength has certain well-defined values $\alpha = (\hbar^2/2m^*a)\sqrt{4(n+1)^2 - 1} \equiv \alpha_n$, for n an arbitrary integer, the difference between $\Phi^{(1)}_{AC} = (2n+1)\pi$ and $\Phi^{(2)}_{AC} = -2\pi - \Phi^{(1)}_{AC} = -(2n+3)\pi$ renders the spin interference destructive and leads to the widest gap because each ring is non-transparent for any value of ka. With the effective mass of InAs $m^* = 0.023m_0$ and a ring radius $a = 0.25\mu$m the smallest value ($n = 0$) of α which can produce total reflection in the chain (and used for the results shown) is $\alpha_0 = 1.147 \times 10^{-11}$eVm.

The rapid oscillations and the square profile of $G(k)$ stem from the fact the chain contains many identical rings. In general, the low transmission values for a single ring become almost zero for a chain with many rings while those values near the maximum 1 remain nearly unchanged. However, this is not exactly true if ka is a half integer. For such a ka we have $\tilde{L}\tilde{L} = -\hat{I}$, where $\hat{I}$ is the identity matrix; consequently, the conductance of a finite chain depends on the parity of N. More precisely, if N is odd G equals the single ring conductance, while

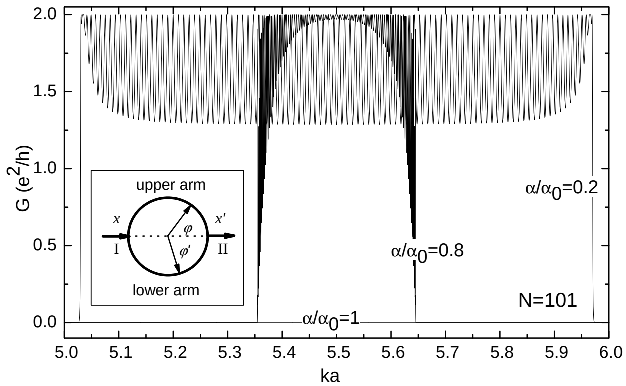

FIG. 1: Conductance through a chain of $N = 101$ rings versus ka for some values of α ($\alpha_0 = 1.147 \times 10^{-11}$eVm). The inset shows the local coordinates (x, x', φ, and φ') pertaining to different regions of the ring.

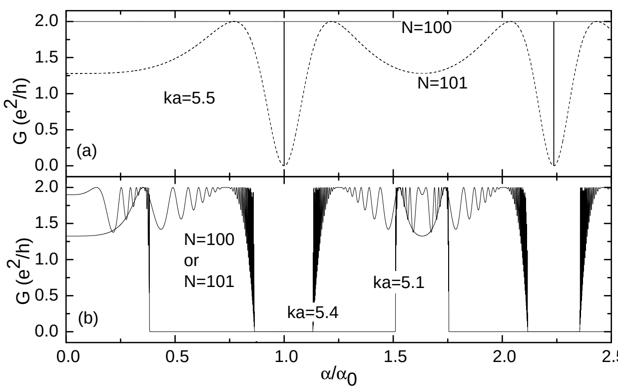

FIG. 2: Figure 2: Conductance versus α for various ka through a chain of $N = 100$ and 101 rings.

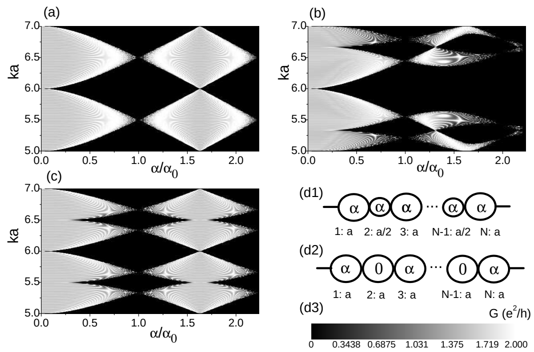

FIG. 3: Contour plot of the conductance through a chain of $N = 101$ rings versus ka and α. In (a) a and α are fixed, in (b) α is kept fixed but a changes as shown in (d1), and in (c) a is fixed while α changes as shown in (d2). Panel (d3) shows the grey color intensity.

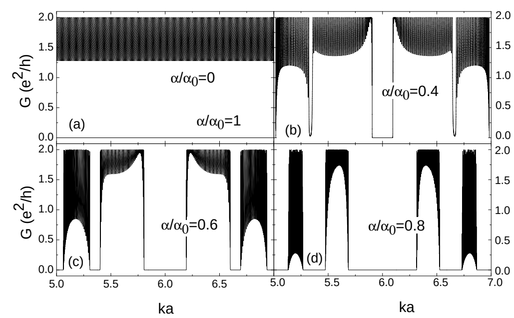

FIG. 4: Conductance versus ka through a chain of $N = 101$ rings pertaining to Fig. 3(c) and the indicated values of α.

if N is even G is a discontinuous function which vanishes only at $\alpha = \alpha_n$, n integer, and otherwise has the value 2. If ka is relatively far from a half integer no difference can be seen between N and $N+1$ for $N = 100$, cf. Figs. 2(a) and 2(b).

Fig. 3(a) is a grey-scale contour plot summarizing the behavior of the conductance G versus ka and α. The darkest regions correspond to ranges of ka and α for which the chain is opaque ($G = 0$) while the white ones correspond to those for which the maximum conductance value is $2e^2/h$ and the chain is transparent. Figure 3(b) shows how a periodic modulation in the ring radius modifies the conductance profile of Fig. 3(a). The radius of the $i^{\rm th}$ ring is given by $a_i = a$, if i is odd, and $a_i = a/2$, if i is even. As seen, the large bright regions along the lines $ka \approx 5.3$ and 6.7, with high conductance, are split apart and produce a full, wide gap along the line $ka = 6$ for a wider range of α. Also, several new gaps appear.

A more regular behavior can be obtained if only α is modulated and the radius is kept fixed. For a chain of $N = 101$ rings having the same radius a but with SOI α changing from ring to ring as $\alpha_i = \alpha$, if i is odd, and $\alpha_i = 0$, if i is even, the resulting $G(k, \alpha)$ is shown in Fig. 3(c). Such a profile for α could be created by applying a gate to only to the i-th ring, i for odd. The contour plot is more symmetric and has definite dark gaps as in the case of a single waveguide.[7] Relative to Fig. 3(a) we see more gaps for constant α and variable ka. Another way of appreciating the results of Fig. 3(c) is shown in Fig. 4 in which $G(k, \alpha)$ is plotted along the lines of Fig. 3(c) at $\alpha/\alpha_0 = 0$, 0.4, 0.6, 0.8 and 1. Notice that in each panel α changes from ring to ring as shown in Fig. 3(d2). As for the widening of the gaps conductance gaps in Figs. 3(c) and Fig. 4 relative to those in Fig. 3(a), it can be understood qualitatively with the help of Eq. (2): upon periodically varying the ring radius or strength α, ω_{so} and the energy levels change from ring to ring thus creating the usual superlattice barriers or wells. Depending then on the incident electron's energy one has the usual gaps or bands in the transmission.

The results presented here are valid for chains of strictly one-dimensional rings. They can be extended to rings of finite width w provided the inequality $w \ll a$ holds and, e.g., an infinite well confinement is assumed along the radial direction. In this case the radial and angular motion are decoupled and the energy levels, given by Eq. (2), are shifted by $\hbar^2 l^2/2m^* w^2$, where l is an integer. The results presented above correspond then to the lowest $l = 1$ mode.

In summary, we studied ballistic electron transport through chains of rings in the presence of SOI, of strength α, and showed that gaps in the conductance, as a function of α and/or the electron's wave vector k, occur due to destructive interferences between electron spins travelling in the clockwise and counter-clockwise directions. In particular, we showed that periodic modulations of α or of the ring radius widen these gaps and produce a nearly square-wave conductance. The full gaps in the conductance plotted in Figs. 2-4 occur whether the incident electrons are spin polarized or not. Accordingly, the results are pertinent to the development of the spin transistor where a spin-dependent and binary conductance output is necessary with as a good control as possible.

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* Department of Physics, Concordia University, 1455 de Maisonneuve Quest, Montréal, Quebec, Canada H3G 1M8

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